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SYSTEMATIC APPROACH TO ESTABLISHING CRITICALITY BIASES

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Systematic Approach to Establishing Criticality Biases

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INTRODUCTION

A systematic approach has been developed to determine benchmark biases and apply those biases to code results to meet the requirements of DOE Order 5480.24 regarding documenting criticality safety margins. Previously, validation of the code against experimental benchmarks to prove reasonable agreement was sufficient. However, DOE Order 5480.24 requires contractors to adhere to the requirements of ANSI/ANS-8.1 and establish subcritical margins.

A method was developed to incorporate biases and uncertainties from benchmark calculations into a k_{eff} value with a quantifiable uncertainty. The method produces a 95% confidence level in both the k_{eff} value of the scenario modeled and the distribution of the k_{eff}s calculated by the Monte Carlo code. Application of the method to a group of benchmarks modeled using the KENO-Va code and the SCALE 27 group cross sections is also presented.

METHODOLOGY

Criticality safety calculations must be adequately benchmarked for the computer code used with representative experiments. Typically a large number of experiments is needed to establish a bias and uncertainty. The method described here can use a minimum of ten benchmarks and provide statistically significant results (see Equation 4). If ten representative benchmarks, (i.e. similar fuel type and form, enrichment, reflector, etc) can not be found, the entire suite of benchmarks is used. In this case, the standard deviation of all of the benchmarks is greater than the standard deviation of ten similar benchmarks. Therefore, conservatism is built into the 95%/95% confidence k_{eff} when the entire library is used. The library of benchmarks should be expanded as the need arises to include at least two benchmarks that are respresentative of the scenario or portions of the scenario (i.e. same reflector material) being modeled.

The benchmark library should first be tested to ensure it represents a normal distribution. A chi-squared test is acceptable but requires a minimum of 50 samples. For smaller libraries, a Kolmogorov-Smirnov (KS) test as described in Reference 1 is applicable. Failure of this test indicates that the benchmarks should be reviewed for accuracy before establishing a bias. If a normal set of representative benchmarks can not be found, the entire set must be used.

Using the statistical analysis outlined below, the computed k_{eff} for a criticality safety calculation can be updated to account for the error in the code to the desired confidence level. The method used as outlined below accounts for the uncertainty in both the mean and the standard deviation of the calculation. The value is dependent on the number of benchmark cases modeled and the resulting statistics in addition to the result of the calculation. First, the code bias,

b_b, for the selected benchmarks is calculated as follows:

$$b_b = \frac{\Sigma D_i}{N_b} \qquad (1)$$

where D_i is the difference between the measured k_{eff} and the value of k_{eff} calculated by the code and N_b is the number of benchmark experiments modeled. The standard deviation, σ_b, is given by

$$\sigma_b = \sqrt{\frac{\Sigma(b_b - D_i)^2}{N_b - 1}} \qquad (2)$$

Finally, the expression for the 95%/95% confidence k_{eff}, termed $K_{95/95}$, is

$$K_{95/95} = k_c + b_b + U(\sigma_b^2 + \sigma_c^2)^{1/2} \qquad (3)$$

where k_c and σ_c are the k_{eff} and standard deviation outputted by the code, and U is the uncertainty multiplier for the required confidences. The standard deviations are included as the sum of the squares as they result from independent events. σ_b is a function of the experimental errors and cross-section errors in modeling the benchmark. σ_c, on the other hand, results from the statistical uncertainty in neutron tracking by the code.

The uncertainty multiplier, U, is a function of, the number of degrees of freedom, df, of the sets of samples as determined using the following equation [2]:

$$df = \frac{(\sigma_b^2 + \sigma_c^2)^2}{\frac{\sigma_b^4}{N_b + 1} + \frac{\sigma_c^4}{N_c + 1}} - 2 \qquad (4)$$

where N_c is the number of stages used to calculate the standard deviation outputted by the code. In the case of the KENO code, N_c is the number of generations run. From df, U is found using the tables for one sided tolerance limit factors for a normal distribution given in Reference 3. Values of U are available for confidence levels of 75% to 99.999% in the standard deviation and a 95% confidence level in the mean. Using an uncertainty factor with 95% confidence in the standard deviation allows one to state that at least 95% of the normal population is less than the $K_{95/95}$ value calculated with 95% confidence, hence the term $K_{95/95}$. Notice as more benchmarks are included, the larger value of N_b increases df which reduces the value of U and subsequently the value of $K_{95/95}$. This fact was used in the decision to require a minimum of 10 benchmarks for application of this method.

As an example, the eleven benchmarks [3,4,5] shown in Table I were used to characterize the SCALE 27 group cross sections and codes for the modeling of N reactor fuel assemblies. The N reactor assemblies consist of uranium metal fuel with an enrichment varying from 0.947 wt% to 1.25 wt% and are of double annular design. The goal of the analysis was to identify criticality safety storage limits for the inner annular fuel rod alone, the outer annular fuel rod alone, and the assembly consisting of the two concentric rods together. The worst case model of the fuel rods used consists of full water moderation and reflection at optimum pitch. The benchmarks modeled to create the code bias

contain single annular uranium metal fuel rods with enrichments from 0.947 wt% to 2.1 wt% with full water moderation and reflection at various pitches. Therefore, all of these benchmarks are very representative of the N reactor fuel and the set provides a good basis for creating a code bias.

The following calculational sequence was employed. First NITAWL was used to calculate the resonance data using the SCALE 27 group cross sections. Next XSDRNPM-S was used to smear the fuel pin and surrounding water cross-sections (i.e. flux weight the cross-sections). Finally the smeared fuel pins were modeled in KENO-Va as a cylinder of the appropriate size for the number of rods. This method was used for both the benchmark and the limiting cases.

As shown by the benchmark results in Table I, the code on average underpredicts the value of k_{eff} for the annular fuel tubes. Therefore a calculated k_{eff} less than 0.98 is not necessarily subcritical. However, by applying the method outlined above to the calculated values, the degree of conservatism is easibly quantified with a 95% confidence level.

The following constants were calculated for these annular fuel benchmarks. To determine the code bias, D_i was first found by subtracting the calculated value of k_{eff} from the measured value of k_{eff} as shown in Table I. Then the code bias was found from Equation 1

$$b_b=\frac{0.1638}{11}=0.0149 \qquad (5)$$

The standard deviation was calculated from Equation 2

$$\sigma_b=\sqrt{\frac{1.731e-03}{10}}=0.0132 \qquad (6)$$

and the variance is given by

$$\sigma_b^2=(0.0132)^2=1.742e-4 \qquad (7)$$

The code statistics were applied to the N reactor fuel results as the following example using a Mark IA inner fuel rod calculation illustrates. KENO-Va calculated a k_{eff} of 0.923 ± 0.00071 for two hundred Mark 1A inner rods at optimum pitch with full water reflection. To apply the bias, df must first be calculated from Equation 4.

$$df=\frac{(0.0132^2+0.00071^2)^2}{\frac{0.0132^4}{11+1}+\frac{0.00071^4}{100+1}}-2 \qquad (8)$$
$$=10.07$$

The KENO run modeled 100 generations; thus this value is used for N_c. To add conservatism, the value of df is always rounded down because a lower degree of freedom results in a higher uncertainty. The corresponding value of U as read from the table in Reference 3 for a df of 10 is 2.815. Thus $K_{95/95}$ for a 95% confidence level in both the mean and the standard deviation is calculated using Equation 3 and is

$$K_{95/95}=0.923+0.0149+2.815(0.0132^2+0.00071^2)^{1/2} \qquad (9)$$
$$=0.975$$

Therefore, in accordance with DOE Order 5480.24, the user can state that there is a

095% probability that the subcritical margin is ≥25 mk with a 95% confidence level.

In addition to the set of benchmarks outlined above, benchmarks sets for a variety of applications including plutonium systems, homogeneous uranium systems, and mixed oxide systems were compiled. Therefore whenever a criticality calculation is performed in the future, representative benchmarks can be chosen 'off the shelf' with no need to run new benchmarks.

CONCLUSIONS

In conclusion, the method described provides an excellent tool for combining benchmark results with calculated values to account for calculational biases in a computer code for a specific problem. With the use of an adequate number of representative benchmarks, the resulting $K_{95/95}$ stands alone as a measure of the safety of the system. The method allows the user to state the margin of subcriticality and the confidence in the value in order to comply with DOE Order 5480.24.

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Table I. Results of N Reactor Fuel Benchmarks				
Pitch (cm)	KENO k_{eff}	KENO bias	D_i $(1-k_{eff})$	$(b_b-D_i)^2$
0.947 wt% Enriched Tubes [4]				
4.52	0.985	0.00073	0.0149	2.321E-9
5.08	0.984	0.00069	0.0160	1.206E-6
1.25 wt% Enriched Tubes [5]				
4.70	0.973	0.00081	0.0265	1.350E-4
5.08	0.980	0.00078	0.0198	2.419E-5
5.33	0.984	0.00070	0.0164	2.397E-6
5.59	0.985	0.00074	0.0153	1.356E-7
6.10	0.987	0.00068	0.0135	1.965E-6
2.1 wt% Enriched Tubes [6]				
6.20	0.958	0.00083	0.0423	7.501E-4
7.11	0.991	0.00091	0.0087	3.884E-5
7.87	1.004	0.00089	0.0035	3.397E-4
8.64	1.006	0.00084	0.0060	4.377E-4
		Total	0.1638	1.731E-3